Swar Sindhu: National Peer-Reviewed/Refereed Journal of Music
A UGC CARE listed Journal
ISSN 2320–7175 | Volume 10, Issue 02, July-December, 2022
http://swarsindhu.pratibha-spandan.org © The Author(s) 2022

INDIAN MUSICAL TREATISES AND MATHEMATICS

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Abstract

Indian music treatises often called *Sangeet Granth* were written by different scholars in Indian music history. These treatises not only describe music and music theory of the time but also give information of the musical thought during the time they were written. The treatises show variety of influences on music prominently of religion, ayurveda, science, mathematics among others. The research in hand is an attempt of highlighting the mathematical influence as mentioned in the musical treatises. The Sanskrit shloks of treatises *Natyashastra* (2nd century approx.), *Brehddeshi* (9th century approx.), *Sangeet Damodar* (14thcentury approx.), *Sangeet Parijat* (17th Century approx.) having mathematical thought behind the musical components are discussed to identify the influence of mathematics in Indian musical treatises.

Keywords: Indian music, Music and Mathematics, Musical treatises, Indian music culture, Indian musical thought

INTRODUCTION

Music and mathematics shares a relation very old, references to which can be found in the writings of Pythagoras, Plato and Aristotle[1,2] Pythagoreans believed that mathematics is the mystical language[3] or the language of God and God has created everything in perfect proportions; Plato believed mathematics as the core of education; Aristotle's writings cover a vast range of topics including music, physics, poetry, theatre, logic, rhetoric, government, politics, ethics and zoology. After studying Musical scales Pythagoras derived a proportion of Musical scale known as "Pythagorean Scale"[4]. Many studies in music are done by different scholars to study the mathematical relationship of notes. The studies included the construction of Musical Instruments[5], relation of the length of string and the note produced[6] etc.

Mathematics is a vast subject which has changed a lot over time. The modern mathematics is considered as an abstract science dealing with shapes, structure, quantity, numbers etc. On the other hand, music deals with combinations and sequences of sounds for purpose of pleasure.

There is abstractness in the representation of the rendering of a raga and mathematicians also use abstract thinking, reasoning and logic to solve mathematical problems. The musicologists of the west have used the language of mathematics to explain music theory. And also the mathematicians were also fascinated by the symmetry, development and harmonization in music due to which the interest to find

out the mechanism working behind music. Music is considered as a reason behind discovery of many laws of nature such as the laws of vibrating strings[7], the harmonics and over tones[8] and many theories related to sound[9] etc.

Considering the study of inter relationship of mathematics and music we can divide the phenomenon of music into two broad categories. One which relates itself to physical world which mathematics can explain; and second the non mathematical psycho physical world. The mathematical world of music covers Alankars, Tanas, Music Scales, Musical intervals, Tihaais, Kayada, Paltas, making of music instruments etc. and the non mathematical side consists of aesthetics, improvisation, Raga Dhyan, Raga Chitra, Swar Lagaav etc.

There are many fields of mathematics which interact with Hindustani Classical Music. Such as combinations and permutations[10], functions[11], series[12], sets[13] etc. which penetrate so deeply in the field of music that mathematics has become an integral part.

“Indian Vedic literature also speaks on the value of Mathematics. ““Ganitam murdhani-Sthitam" means mathematics stands at the top is a quote from Vedic literature on the importance of mathematics in other Vedic sciences. The auxiliary science of Vedic prosody deals with the formation of various metres for melody and rhythm. Pingala's Chandahsutra (about 200 BC) is taken to be a Vendanga although it is a late work dealing with Vedic as well as non Vedic metres. The mathematical importance of the work likes in the fact that it has material on the theory of permutations and combinations.”[14]

Indian Classical Music, as it was confined to Brahmanic rituals during Vedic & Post Vedic period[15], for entertainment purposes during Mughal period[16], has lost its importance during British period is now rejuvenating with high statutory with applications in Medicine[17], Psychology[18], Enhancement in Plant Growth[19] etc. along with the Entertainment and religious purposes. Time to time many musical Granthas[20] were written on Hindustani Classical Music. The Musical Granthas, the very basis of Hindustani Classical Music, also deals with the Mathematics of Music. The Study of these led to the discovery of *Shlokas,* using higher level mathematical calculations as a tool for construction of Music Theory. Some of the ideas described in musical granthas are mentioned here.

MATHEMATICAL FORMATION OF INDIAN *TALAS*

Tala is literally translated as Rhythmic cycle or ‘Metric cycle’[21]. It is an inseperable part of Indian music. All the forms of music, may it be vocal, instrumental or dance are bound to metre. The Indian concept of Tala is very unique. All music is performed

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in a particular rhythm. When the 'number' of rhythmic beats (called *Matra* in Indian Music) occuring in one cycle becomes the measuring entity, it forms the tala.

DIVISION OF TALAS INTO FIXED BASIC UNITS

Natyashastra treatie define tala as:

Taalo Ghana Iti Prokta Kala-Pata-Layanvita
Kalastasya Pramaan Vai Vigyeayang Talyoktribhi[22]

Tala as made up of *Kala* (time), *Pata* (sound/syllable of instrument) and *Laya* (rhythm) measured on Ghana (idiophone) instrument.

Pingle (1898) defines tala as:
"Tala (time), which is an important factor in every system of music, regulates the relative durations of musical sounds, and as such seems to have considerably engrossed the Indian mind."

The ancient concept of tala is quite scientific. Tala is basically measuring the time. As time is prevailing in the universe without any start or end, It cannot be measured subjectively. But objectively, it can be measured with the help of days, hours, minutes, seconds etc. Similarly to measure the time in music, beat duration in *Tala* is used. The rhythmic beats (*Matras*) are understood in four ways according to the beat duration. Musical treatises give names to the beat duration of the talas. i.e. the time space is coded.

Sankshepto Nigdita Athaishan Lakshanan Yatha I
Ardhmatran Drutan Gyayemekmatran Laghu Smritmam I
Dvimatrantu Gurugyayean Trimatrantu Plutan Matam I[23]

According to the shloka, the half-beat is named as 'drut', one beat as 'laghu' two beats as 'Guru', and three beats as 'Pluta'.

To understand this, we can assume a continuity of claps at the interval of one second each. Then each clap is termed as 'laghu'. Then taking the interval of one second as constant, if there are two claps in one second i.e. half second time for each clap then the clap is termed as 'drut'. Similarly if one clap is missed after each clap, the time interval after each clap is two seconds. Then the clap is termed as 'Guru'. And if each clap is sounded after three seconds then it is termed as 'Pluta'.

ORDERING OF THE UNITS OF TALA

With different combinations, sequences and lengths of these different time units, talas were composed. In all this process the time is divided into mathematical units and

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then these units are arranged systematically to form different talas. Representing mathematically, we can write that the time-space is divided into proportions of x, 2x, 3x and 1/2x is named as Laghu, Guru, Pluta and Drut respectively which are then forming different talas by creating different sequences of these four time periods. The use of these proportional sizes of the time and their different sequential arrangements to form different talas is an example of combinatorics and Algebra.

Hansak Tala consists of 'Laghu Guru Laghu' and is of 12 beats according to the shloka in Sangeet Damodar.

Dwadashaaksharpaada: Syaat Sa Chalapashubhakrit Prabho: I
Hansake Ch Rase Veere Giyate Shekharo Dhruva: II
Laghurgururlaghuryatra Sa Taalo Hansak: Smrita: II[24]

It means that the *Shekhar Dhruva* is performed with emotions of heroism or bravery and it is bound to the rhythm of 12 time units with the sections arranged in the sequence Laghu-Guru-Laghu i.e. time spacing of 'x, 2x, x' repeats until the completion of 12 time units.

Hence mathematically, the *Tala* becomes:

$$(x + 2x + x) + (x + 2x + x) + (x + 2x + x) = 12x = 12 \text{ time units.}$$

Pt. Shubhankar gives the names of 60 talas[25], as given by Bharata in 'Natya Shastra', also describing the arrangement of Guru, Laghu etc. for each tala.

Pt. Damodar, in sanskrit verses of the treatise mentions sixteen talas from 11 to 26 beats with their sequence of sections. In some of the talas, the sequence is not mentioned. These talas have a special heading of '*Aparan Niyaman Vina*' which means other without rule. In the description of these talas it is mentioned that they are made up of the parts of 1/2, 1, 2 and 3 time units. Similarly all the sixty talas are described using the same unit sizes arranged in different order. Some talas like Garg Tala, Krida Tala, Laghushekher Tala etc. have special heading 'Other without rule'

Aprang Niyamang Bina
Kridataale Dhruvsh Syat Paade Panchshabadakshar:
Nirmal: Shringarrase Samriddhifalvardhan
Ek Ev Pluto Yatra Kridataal: sa Kathyate[26]

In the above shloka for Krida Taal the heading *Aprang Niyamang Bina* means other without rule i.e only the total number of beats is mentioned.

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MATHEMATICS IN THE FORMATION AND CALCULATION OF *TANAS*

Matang Muni in *Brehdeshi* (around10th century CE)[27] explains the formation of 5040 Tanas from the 7 notes of the scale in the verses 57,58 of chapter 1, section 5[28]. It is described that with two numbers, maximum number of combinations is two i.e. 'a b' and 'b a' {Mathematically '2 Factorial'}; and with three numbers is two combinations with two numbers multiplied by 3 which is equal to 6 {Mathematically '3 Factorial'}; number of combinations with 4 numbers is 4 multiplied by number of combinations formed with 3 numbers which is equal to 4 x 6 = 24 {Mathematically '4 Factorial'}; and number of combinations with 5 numbers is 5 multiplied by number of combinations formed with 4 numbers which is equal to 5 x 24 = 120; number of combinations with 6 numbers is 6 multiplied by number of combinations formed with 5 numbers which is equal to 6 x 120 = 720; and number of combinations with 7 numbers is 7 multiplied by number of combinations formed with 6 numbers which is equal to 7 x 720 = 5040 = 7 x 6 x 5 x 4 x 3 x 2 x 1= 7 factorial.[29]

By different combinations and permutations of the seven notes and the number 5040 is equal to '7 factorial'. The method of calculating the maximum number of permutations formed by each of 1, 2, 3, 4, 5, 6, 7 notes is given.

The description shows the knowledge of the calculation of number of different combinations which can be formed with any natural number of different notes i.e. the total number of combinations with n number of different notes is equal to n multiplied by the number of combinations formed by (n-1) number of notes, which is equal to **n** multiplied by **(n-1)** multiplied by **no. of combinations with (n-2) number of notes**… and so on. Extending the equation further and further we get n x (n-1) x (n-2) x (n-3)..... x 1 which is equal to mathematical n factorial. Although the musical treatise restricts the calculations of 7 notes of Octave or 7 numbers (mathematically speaking), the concept of the calculation of the maximum number of combinations with 'n' numbers was clear to Ancient Music Scholars.

MATHEMATICAL MEASUREMENT IN THE DIVISION OF OCTAVE

As musical notes are not absolute but relative to the base note hence singular notes relating them to the frequency was not discussed in musical treaties but the relation of the pitch of the notes corresponding to the length of string were discussed. Pt *Ahobal* (Author of Sangeet Parijat) and Pt. *Srinivas* (Author of Raga Tatva Vibodh) explained the attainment of the notes of Octave through mathematics by relating the pitch of note to the length of string. Among the musicologists of medieval age, Pt. *Ahobal* has a very important place. The important is due to his demonstration of the notes of Octave on stringed instrument with the help of mathematics. In this experimental

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demonstration of notes, the subjective experience of the musician is completely excluded and the notes are demonstrated objectively and experimentally on the basis of the traditional placement of frets and the knowledge of musicians.

Swaragyanavihinebhyo Magodhya Darshito Mya[30]

According to ancient musicologists, a musician (by rigorous practice under a guided musician) must be able to identify the notes of the scale only through hearing but they demonstrated in their treatises the method of identification of notes on the length of string for those who are devoid of musical knowledge and the knowledge of notes.

Swarasya Hetubhutaya Veenashrashrushtavata: I
Tatra Swaravibodhartha Syanlaskhanmuchayate II[31]

The stringed instrument veena produces the musical notes that apart from listening to the notes, they are also visualised in their positioning on the string of veena. Because of this reason the notes of Octave are demonstrated on the veena with string length of 36 inches. Pt. Ahobal and Pt. Shrinivas explains the placement of notes on the string mathematically by relating the pitch of note produced in terms of the Length of the vibrating part of string which is again calculated mathematically.

The Notes of the scale in terms of the length of string[32] as described by Pt. *Ahobal* in verses 313-327 and Pt. *Srinivas* are tabulised as under:

SN	**Western Names of Notes if First Note is considered as 'C'**	**Notes of Modern Octave**	**Corresponding notes of Pt. Ahobals and Pt. Srinivas Octave**	**Length of String from Taargahan on 36 Veena String**
1	C	Shadaj	Shadaj	0 Inches
2	C#	Komal Rishabh	Komal Rishabh	$2\frac{2}{3}$ Inches
3	D	Shudh Rishabh	Shudh Rishabh	4 Inches
4	D#	Komal Gandhar	Shudh Gandhar	6 Inches
5	E	Shudh Gandhar	Teevra Gandhar	$7\frac{1}{3}$ Inches
6	F	Shudh Madhyam	Shudh Madhyam	9 Inches
7	F#	Teevra Madhyam	Teevratom Madhyam	$10\frac{8}{9}$ Inches
8	G	Pancham	Pancham	12 Inches
9	G#	Komal Dhaivat	Komal Dhaivat	14 Inches
10	A	Shudh Dhaivat	Shudh Dhaivat	$14\frac{2}{3}$ Inches
11	A#	Komal Nishaad	Shudh Nishaad	16 Inches
12	B	Shudh Nishaad	Teevra Nishaad	$16\frac{8}{9}$ Inches
13	C	Taar Shadaj	Taar Shadaj	18 Inches

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The accuracy of these calculations can be questioned but what is sure is the attempt to mathematically corelate the Frequency or the Pitch of Note to the Length of String.

FUNCTIONS AND SET IN FORMATION OF ALANKAARS

Alankaars are the base of learning Indian music. For the notes of '*Thata*'[33] (a category of the set notes defining the form of notes to be used in the ragas that fall under it) arranged in ascending order of frequencies, a particular pattern is selected and then applied to the notes of the *Thata* starting from the first note of the *thata* forming the first phrase of the *alankaar*. The same pattern is then applied starting from the next note the *Thaat* forming the second phrase. The process continues for all the notes of the *Thata* to form a full sequence of phrases. The applied pattern is always mathematical in nature and can be called as a function which is applied to the basic set of notes.

Aroh : Sa Re Ga Ma Pa Dha Ni Sa˙
Avroh : Sa˙ Ni Dha Pa Ma Ga Re Sa

The formation of all the phrases of a particular *Alankaar* becomes an arranged sequence of a particular pattern developed starting from Middle '*Sa*' to *Taar 'Sa'*. Mathematization of the pattern can be easily understood with the help of the example of the very first Alankaar. The Mathematical function A= $[x_n]_{n=1}^{n=8}$ {for n>7, $x_n = x_{n-7}$, till 0<n<8}.

Here the base Set from which the values of x_1, x_2, x_3 are taken is the set of notes of Aroh and Avroh of First Alankaar of a particular Thata

i.e. the set
X= [X_1=Sa; X_2 = Re; X_3 =Ga; X_4 =Ma; X_5= Pa : X_6 = Dha; X_7= Ni]
{For X>8, X_n=X_{n-7}} until X_1 >= X_n >= X_7

As per sayings of musicians, infinite number of *Alankars* can be devised and formatted using the notes of the octave. Verses (called Annuched in the treatise) 70-102 of Brehdeshi describes the structure of 33 Alankars explaing their construction mathematically. One of the verse is mentioned below.

Dvayo: Swaryorbahuna Ch Swarana Samkalgamagamachaturdarshkal: Prenkholita: I

Yatha- SaRe ReSa ReGa GaRe GaMa MaGa MaPa PaMa PaDhaDhaPa DhaNi NiDha NiSa SaNi (Iti) Prenkholita: I[34]

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Verse 81of Brehdeshi explains the formation of *Prenkholita* alankaar by using two or more notes at a time. Using them in ascending and descending order of frequencies alloting same time to each note. The minimum duration with the use of 2 notes at a time will be 14 beats for the 7 notes of Indian music. The formation is just written but explained. This alankaar is formed by using the pattern {x 'x+1' 'x+1' x} applied to the notes of octave. If the seven notes are assigned the numbers 1, 2, 3, 4, 5, 6, 7 then the first combination of sequence is Sa Re Re Sa corersponding to 1, 2, 2, 1. The Next combination becomes Re Ga Ga Re corresponding to 2, 3, 3, 2 for x = 2 the combination Ga Ma Ma Ga corresponding to 3, 4, 4, 3 corresponding to x = 3 is the next term in the sequence. The mathematical function can be defined as:

A= $[x_n x_{n+1} x_{n+1} x_n]_{n=1}^{n=8}$ {for n>7, $x_n = x_{n-7}$, till 0<n<8}.

The base Set remains the same for the values of x_1, x_2, x_3 i.e the Aroh and Avroh of First Alankaar of a particular Thata

i.e. the base set is

X= [X_1=Sa; X_2 = Re; X_3 =Ga; X_4 =Ma; X_5= Pa : X_6 = Dha; X_7= Ni]
{For X>8, X_n=X_{n-7}} until X_1 >= X_n >= X_7

Simlarly a priliminary Alankaar in Indian music

Aroh: Sa Re Ga, Re Ga Ma, Ga Ma Pa, Ma Pa Dha, Pa Dha Ni, Dha Ni Sa˙
Avroh: Sa˙ Ni Dha, Ni Dha Pa, Dha Pa Ma, Pa Ma Ga, Ma Ga Re, Ga Re Sa

Can be mathematically written in the form of function as

A= $[x_n x_{n+1} x_{n+2}]_{n=1,}^{n=7}$ applied on the Aroh and Avroh of First Alankaar

Here the base Set from which the values of x_1, x_2, x_3 are taken is the set of notes of Aroh and Avroh of First Alankaar of a particular Thata

i.e. the set
X= [X_1=Sa; X_2 = Re; X_3 =Ga; X_4 =Ma; X_5= Pa : X_6 = Dha; X_7= Ni]
{For X>8, X_n=X_{n-7}} until X_1 >= X_n >= X_7

CONCLUSION

Mathematics is embedded in each and every aspect of Indian music. Mathematical ordering, measurement, division, calculations play important role in theorization, explanation, practical understanding and composition in classical music. The Indian music treatises, written in the form of sanskrit shloks, are storehouse of knowledge. These explain about the theory of subject along with relating different disciplines with

each other. The Indian musical treatises relates music with mathematics in the formation of talas, division of talas into fixed basic units, ordering of the units of tala, formation and calculation of tanas, measurement in the division of octave, functions and set in formation of alankaars etc. The musical treatises can be furthur studied to explore the relationship of music with different disciplines.

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